Speed of Sound Using Lissajous Figures

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Demonstration of the speed of sound in air is a classic component of the general physics curriculum. This demonstration uses a sine wave from an audio oscillator and the same signal picked up by a movable microphone to produce Lissajous figures and determine the speed of sound.

Early references are available in the *American Journal of Physics* and *The Physics Teacher* describing basic methods to carry out this measurement.[1,2] Several interesting early papers in the *AJP* discuss thoroughly the theoretical basis for the speed of sound.[3,4,5]

A visually elegant technique for determination of the speed of sound involves Lissajous figures. Apparently first mentioned in an extensive Russian publication,[6] this technique was the subject of a short 1964 paper in *TPT*.[7] Several laboratory descriptions that can be accessed on the web describe aspects of this technique. An extensive German manual from Universität-Gesamthochschule Siegen[8] describes the experiment using a Kundt's tube, and two descriptions from a University of Dortmund manual entitled *Lernwerkstatt Physik Praktikum* use this technique for determination of the speed of sound in various gases by placing the system in a container in which the gases are confined.[9] The University of Melbourne Lecture Demonstration Online Manual briefly mentions this demonstration,[10] and a more detailed online laboratory manual at Dartmouth describes two techniques for determination of the speed of sound, one of which (called "Traveling Wave Measurement") involves using Lissajous figures to determine the speed of sound in a narrow Kundt's tube.[11] This experiment is also included in the University of Maryland online physics demonstration library.[12]

This paper will update the discussion and provide a beautiful but simple technique for determination of the speed of sound using Lissajous figures that can be readily shown as a lecture demonstration. We have used this technique in our Physics of Music class for nonscience students, both as a way to measure the speed of sound and to introduce discussion of some basic concepts such as phase differences between waves.

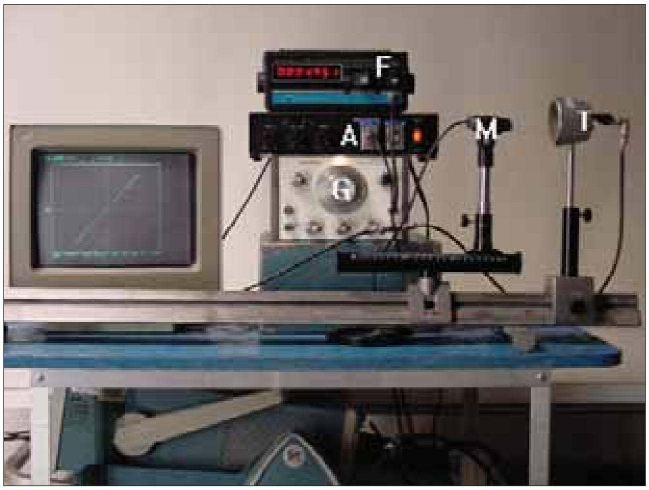

Fig. 1. Apparatus used in the measurement of the speed of sound using Lissajous figures.

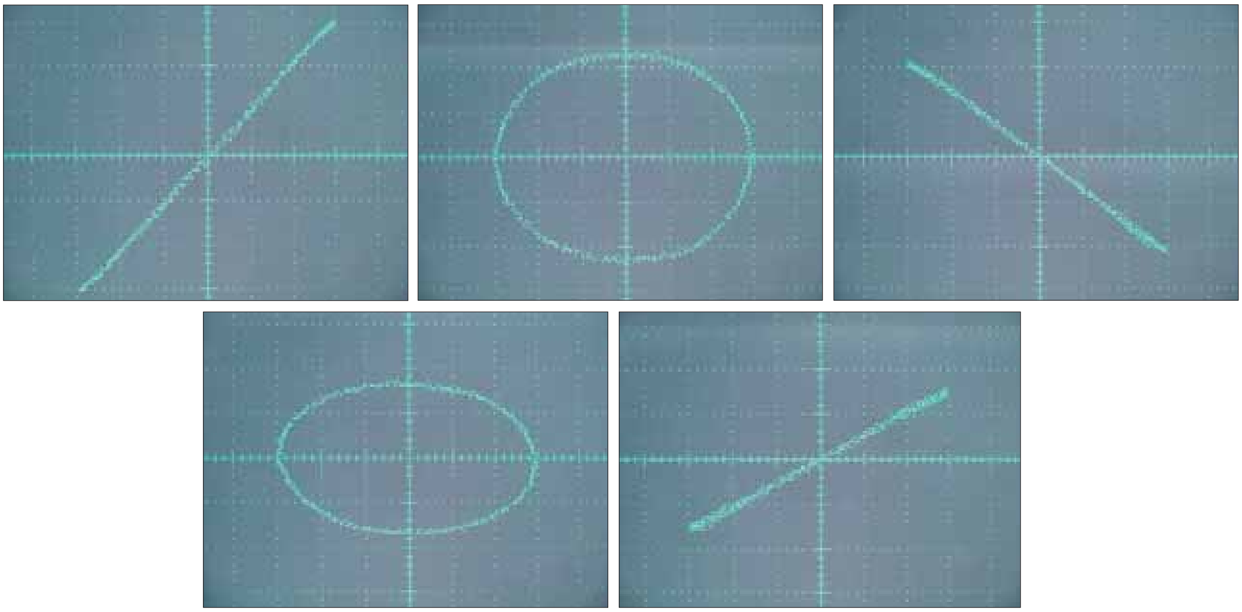

Fig. 2. Sequence of Lissajous figures obtained as the microphone is moved away from the loudspeaker.

Experimental Setup

The apparatus used in this demonstration is shown in Fig. 1. The sinusoidal output from a wave generator, G, is input simultaneously into a tweeter, T, and the horizontal axis of the oscilloscope. The signal picked up by the microphone, M, is fed into an amplifier, A, and then to the vertical axis of the oscilloscope. The oscilloscope, resting on a shelf below the apparatus, is a Tektronix digital scope with an EGA-compatible output that is displayed by the computer monitor on the platform next to the experimental electronics. To get a good measurement of the frequency, an external frequency meter, F, is connected to the trigger output of the wave generator. The setup table as photographed was assembled on top of an old Tektronix scope-mobile and can be rolled into the classroom. For large lecture halls the EGA output from the oscilloscope is fed into an RGB converter and then to a video projector providing a 12-ft diagonal rear-projection display, so several hundred people can easily view the oscilloscope output. The smaller sliding optical rail makes it easier to move the microphone and set it to one of the desired points in the evolving Lissajous pattern.

In Fig. 1, the microphone has been positioned such that the signals from the loudspeaker and the microphone are in phase, resulting in a diagonal line Lissajous figure. If the two amplitudes are equal, that line will be at a 45° angle with respect to the *x*- and *y*-axes. As the microphone is moved away from the loudspeaker, its signal lags in phase with respect to that applied to the loudspeaker, and the Lissajous pattern changes, as seen in the sequence of photographs in Fig. 2, running from upper left to lower right.

If the amplitude of the microphone signal were to remain constant, the line figure obtained when the two axes are in phase or out of phase would be at an angle of 45°. A relevant problem for your students to discuss is why the slope of the line for the in-phase and out-of-phase figures decreases as you pull the microphone away from the loudspeaker, and why the intermediate patterns are ellipses rather than circles. The slope can be maintained at 45° by increasing the gain of the vertical signal as the microphone is withdrawn, but we selected to let the amplitude decrease in order to keep the experiment simple.

Discussion

The speed of sound in an ideal gas S is given by the equation[13]

$$S = \sqrt{\gamma \mathrm{R} T / M}, \qquad (1)$$

where γ is the adiabatic constant, T is the absolute

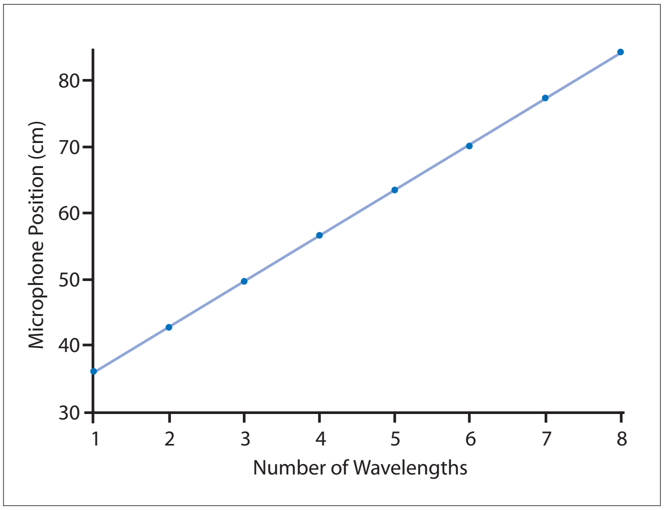

Fig. 3. Graph for obtaining the wavelength.

temperature in kelvins, M is the molecular weight of the gas, and R is the gas constant per mole, 8.314 J/mol K. *A Physicist's Desk Reference*[13] gives the value of 331.45 m/s for the speed of sound in dry air at 0°C. The average molecular weight of dry air is 28.95 g/mol, and the adiabatic constant γ is 1.4, so this equation becomes

$$S = 20.05\sqrt{T}\ \text{m/s}. \tag{2}$$

A less accurate, but commonly used linear approximation for the speed of sound in air around room temperature is

$$S \sim 331.45 + 0.61\ T_c\ \text{m/s}, \tag{3}$$

where T_c is the air temperature in degrees Celsius.

The speed of sound can be obtained from our data as:

$$S = \frac{d}{t} = \frac{\lambda}{T} = \lambda f, \tag{4}$$

where the wavelength λ is related to the distance the microphone is moved, T is the period, and f is the frequency of the wave. The microphone moves exactly one wavelength in changing the Lissajous figure from the initial to the final configuration shown in Fig. 2. A more accurate value for λ can be obtained by determining the slope of the graph of microphone position versus number of wavelengths.

Figure 3 shows a plot of typical results. We moved the microphone seven full cycles, obtaining an average wavelength of 6.93 cm using a frequency of 4949.5 Hz as read from the frequency meter. This gave the experimental value S = 344.5 m/s. Using Eq. (2) above, with the measured room temperature of 17.2°C, we obtain a theoretical value of 341.3 m/s. The air had about 40% relative humidity, which would be expected to increase the theoretical speed of sound very slightly. This result is more than sufficient as a lecture demonstration to show the important features of the experiment.

With minimal care, the measurement will be within less than 5% of the value calculated using Eq. (2). By making measurements of the "in-phase" positions over several pattern cycles, you can improve the accuracy and notice any systematic variations. Small inconsistencies seem to occur due to changes in the microphone pickup of reflections in the room, and the pattern is so sensitive that it may change as you move around near the apparatus. (This sensitivity is one reason for doing the experiment in a tube.) You can minimize this effect by selecting a high frequency

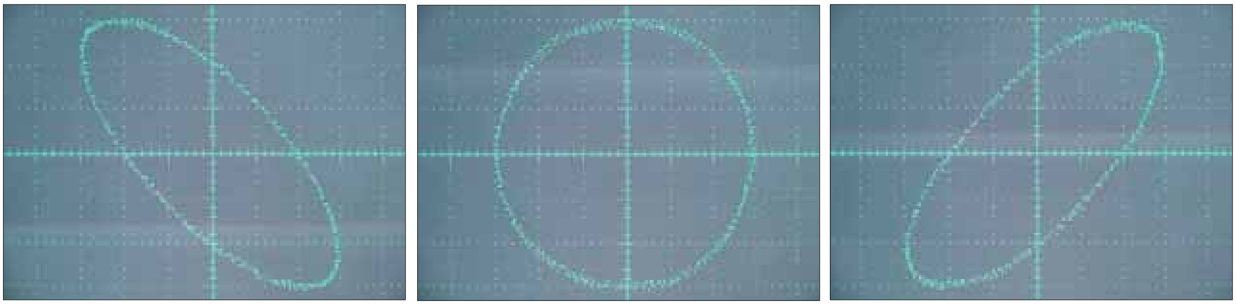

Fig. 4. Lissajous patterns for the question in the text.

and by aiming the sound toward a distant wall or a region where there are few coherent reflections. If care is taken, the accuracy will be better than 1%.

When the air between tweeter and microphone is heated, for example with a heat gun, a change in Lissajous pattern is observed. The sensitivity of this setup allows at least a semi-quantitative verification of the temperature dependence of S. If any pattern is set up on the oscilloscope and the microphone is then moved away from the speaker, the original pattern can be at least partially restored with the heat gun, showing that the speed increases when the temperature increases. Our heat gun produces a stream of air at approximately 110°C with a diameter of about 3 to 4 cm, about half the wavelength of the sound used. This is sufficient to move the vertical signal ahead in phase about 45°. The result is that the diagonal line seen when the two signals are in phase is changed into a diagonal ellipse. (A 90° phase difference would produce a circle.)

This experiment can be used as a question for your students. Starting with the Lissajous pattern shown in the first photograph of Fig. 2, the microphone is moved away from the loudspeaker and the vertical amplitude increased slightly to produce the second pattern in Fig. 4. If the heat gun is aimed across the sound path from the speaker to the microphone, which of the three photographs in Fig. 4 will most nearly resemble the Lissajous pattern produced?

In order to obtain the answer, you must either do the experiment yourself or go to Question #157 of the Physics Question of the Week website.[14]

We believe that this demonstration is very effective both because it is esthetically beautiful and because it allows many opportunities to involve students during the presentation.

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